

# Performance Comparison between Traditional and Gray-mapped 16-QAM Scheme with OFDM in both AWGN and Rayleigh Fading Channel

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**Abstract**—Orthogonal Frequency Division Multiplexing (OFDM) using M-ary Quadrature Amplitude Modulation (QAM) is a very common approach in multicarrier communication. With the increasing demand of multimedia communication, the concept of OFDM was introduced. In this paper, we have shown a comparative analysis between the traditional binary mapped 16-QAM and gray mapped 16-QAM scheme with OFDM in both AWGN and Rayleigh fading channel. We have chosen the parameters SER vs SNR for binary mapping and BER vs SNR for Gray mapping for the performance analysis as these two are the most important parameter for any wireless communications. The selection of 16-QAM scheme was made to reduce the complexity of higher order QAM constellations which are more susceptible to noise.

**Index Terms**—OFDM, 16-QAM, Gray Mapping, AWGN, Rayleigh Fading.

## 1 INTRODUCTION

THE radio environment is harsh, due to the many reflected waves and other effects. Demands of the wireless multimedia broadband system are anticipated within both public and private sector. OFDM, a multi carrier modulation technique is a promising candidate that eliminates the need of complex equalizers. For increasing the capacity of the system we have selected Quadrature Amplitude Modulation (QAM), which is a introduced by the fading channels make low error transmission of QAM difficult to achieve, unless procedures are introduced at both the transmitter and the receiver to combat the fading. Some recursive methods of improving the SER of QAM along with OFDM using various form of coding on an increased symbol set are established [1]. This means that the data throughput, symbol rate and transmission power are unaffected, although the transmitter and receiver are made considerably more complex. In this paper, we have implemented simple Gray mapping [2] technique and showed the improved SNR performance. The normal binary ordering is the general numeral system that represents numeric values using two symbols, i.e. 0 and 1 where as the reflected binary code, also known as Gray code after Frank Gray, is a binary numeral system where two successive values differ in only one digit [3]. In our paper we have simulated the SER vs. SNR and BER vs SNR

curve for AWGN channel as well as Rayleigh fading channel. In the deliberations Rayleigh, rather than the less severe Rician, fading channels are considered to obtain worst case performance estimates of mobile radio communications [4].

## 2 QAM TRANSMISSION OVER DIFFERENT CHANNELS USING OFDM

### 2.1 AWGN CHANNEL MODEL

The transmitted waveform gets corrupted by noise, typically referred to as Additive White Gaussian Noise (AWGN).

Additive: As the noise gets 'added' (and not multiplied) to the received signal.

White: The spectrum of the noise is flat for all frequencies

Gaussian: The values of the noise ( $\eta$ ) follow the Gaussian probability distribution function,

$$P(x) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(x-\mu)^2}{2\delta^2}} \text{ with } \mu = 0 \text{ and } \delta^2 = \frac{N_0}{2}.$$

### 2.2 RAYLEIGH FADING MODEL

In a multipath environment, it is reasonably intuitive to visualize that an impulse transmitted from transmitter will reach the receiver as a train of impulses.

Let the transmitted bandpass signal be

$$x(t) = \Re\{x_b(t)e^{j2\pi f_c t}\} \quad (1)$$

Where  $x_b(t)$  is the baseband signal,  $f_c$  is the carrier frequency and  $t$  is the time.

As shown above, the transmit signal reaches the receiver through multiple paths where the  $n^{th}$  path has an attenuation  $\alpha_n(t)$  and delay  $\tau_n(t)$ .

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The received signal is

$$r(t) = \sum_n \alpha_n(t) x[t - \tau_n(t)] \quad (2)$$

Plugging in the equation for transmit baseband signal from the above equation,

$$r(t) = \Re\{\sum_n \alpha_n(t) x_b[t - \tau_n(t)] e^{j2\pi f_c [t - \tau_n(t)]}\} \quad (3)$$

The baseband equivalent of the received signal is  $r_b(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} x_b[t - \tau_n(t)]$

$$= \sum_n \alpha_n(t) e^{-j\theta_n(t)} x_b[t - \tau_n(t)] \quad (4)$$

where  $\theta_n(t) = 2\pi f_c \tau_n(t)$  is the phase of the  $n^{th}$  path.

The impulse response is

$$h_b(t) = \sum_n \alpha_n(t) e^{-j\theta_n(t)} \quad (5)$$

The phase of each path can change by  $2\pi$  radian when the delay  $\tau_n(t)$  changes by  $\frac{1}{f_c}$ . If  $f_c$  is large, relative small motions in the medium can cause a change of  $2\pi$  radians. Since the distance between the devices is much larger than the wavelength of the carrier frequency, it is reasonable to assume that the phase is uniformly distributed between 0 and  $2\pi$  radians and the phases of each path are independent [5]. When there are large numbers of paths, applying Central Limit Theorem, each path can be modeled as circularly symmetric complex Gaussian random variable with time as the variable. This model is called Rayleigh fading channel model. A circularly symmetric complex Gaussian random variable is of the form

$$Z = X + jY \quad (6)$$

,where real and imaginary parts are zero mean independent and identically distributed Gaussian random variables. For a circularly symmetric complex random variable  $Z$ ,

$$E[Z] = E[e^{j\theta} Z] = e^{j\theta} E[Z] \quad (7)$$

The statistics of a circularly symmetric complex Gaussian random variable is completely specified by the variance

$$\sigma^2 = E[Z^2] \quad (8)$$

The magnitude  $\frac{\sigma^2}{2} |Z|$  which has a probability density

$$p(z) = \frac{z}{\sigma^2} e^{-z^2/\sigma^2}, z \geq 0 \quad (9)$$

is called a Rayleigh random variable. This model, called Rayleigh fading channel model, is reasonable for an environment where there are large numbers of reflectors.

### 3 16-QAM

It is a one type of M-ary QAM where  $M=16$ . In 16-QAM modulation scheme we can send ( $k=\log_2 M = \log_2 16 = 4$ ) 4 bit information per symbol [6]. Now, here we consider that the alphabets used for a 16-QAM is  $\alpha_{16QAM} = \begin{pmatrix} \pm 1 + \pm 1j; \pm 1 + \pm 3j; \\ \pm 3 + \pm 3j; \pm 3 + \pm 1j \end{pmatrix}$  [7]. In Fig.1 we have shown 16-QAM constellation diagram and in Fig. 2 we have shown 16-QAM with gray coded bit mapping.

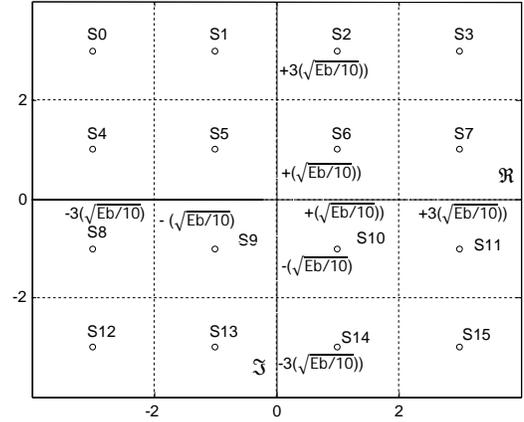


Fig. 1 : 16-QAM constellation

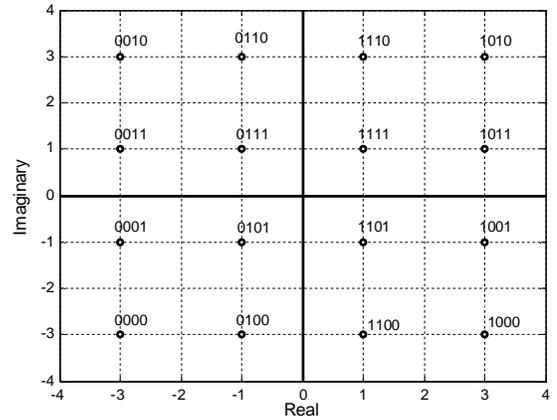


Fig. 2: Gray coded bit mapping for 16-QAM.

### 4 OFDM

The concept of using parallel data transmission by means of frequency division multiplexing (FDM) was published in mid 60s [8,9]. Some early development can be traced back in the 50s. A U.S. patent was filled and issued in January, 1970. The idea was to use parallel data streams and FDM with overlapping subchannels to avoid the use of high speed equalization and to combat impulsive noise, and multipath distortion as well as to fully use the available bandwidth.

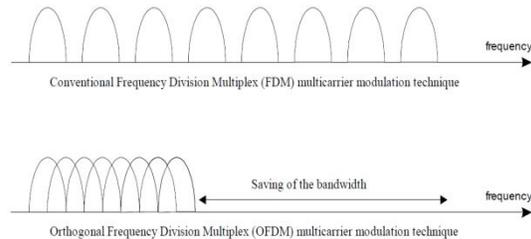


Fig. 3: Comparison of the bandwidth utilization for FDM and OFDM

The initial applications were in the military communications. In the telecommunications field, the terms of discrete multi-tone (DMT), multichannel modulation and

multicarrier modulation (MCM) are widely used and sometimes they are interchangeable with OFDM. In OFDM, each carrier is orthogonal to all other carriers. However, this condition is not always maintained in MCM. OFDM is an optimal version of multicarrier transmission schemes.

**4.1 OFDM DESIGN CONSIDERATIONS**

One of the design considerations for OFDM includes sensitivity to frequency offset, where frequency offset correction must be performed in the receiver. OFDM is also sensitive to oscillator phase noise, so a clean and stable oscillator is required. The waveform also has a large peak to average ratio, so the amplifier must be backed off from saturation which reduces the transmit power efficiency. Another design consideration is FFT and inverse FFT implementation to optimize latency with performance. OFDM waveforms experience inter-symbol interference (ISI) and inter-channel interference (ICI) due to multipath in the RF channel through which the signal is propagated. They can use a guard interval created by a cyclic prefix to mitigate the problem. The cyclic prefix is made by replicating part of the OFDM time-domain waveform from the back to the front. The duration of the guard period is longer than the worst-case delay spread of the multipath environment, so multipath delays up to the guard time will not cause ISI and the subcarriers will remain orthogonal for multipath delays up to the guard time, which eliminates ICI. To reduce spectrum splatter, the OFDM symbol is multiplied by a raised-cosine window before transmission to more quickly reduce the power of out-of-band subcarriers; however, the roll-off factor reduces delay spread tolerance. The parallel transmission of data over many carriers helps protect against frequency-selective fading, where some subcarriers may be degraded and others are unaffected. Forward error correction coding is also used to provide redundancy so that the correctly received bits can be used to correct errors in poorly received channels. Bursts of errors in a given time interval or over a given frequency band are reduced in the time domain by time staggering the coded bits and reduced in the frequency domain by interleaving the coded bits to specific subcarriers.

**4.2 OFDM WAVEFORM TRANSMISSION**

OFDM waveforms consist of multiple modulated orthogonal sub-channel RF carriers multiplexed into a single composite wideband radio signal. Fig. 4 shows an FFT (Fast Fourier Transform) based implementation of an OFDM system, where an Inverse FFT (IFFT) is used to generate the waveform and a forward FFT is used to receive it. Using the IFFT and FFT is very practical and eliminates the need to separately modulate and demodulate the many different OFDM subcarriers. By implementing an IFFT at the transmitter and an FFT at the receiver, OFDM converts an ISI-inducing channel with Additive White Gaussian Noise (AWGN) into many parallel ISI-free subchannels with gains equal to the channel's FFT frequency response.

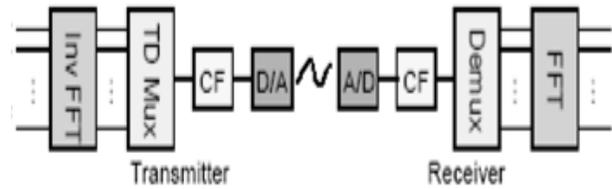
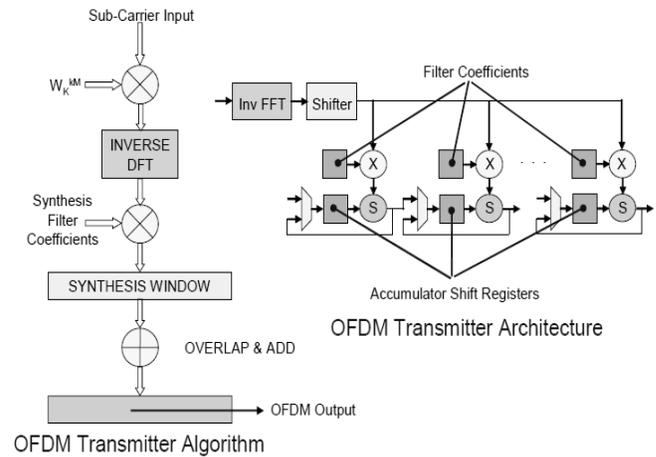


Fig. 4: FFT-Based OFDM System

Each subchannel can be easily equalized by a single-tap equalizer using scalar division. To avoid inter-block interference (IBI) between successive blocks, a cyclic prefix is inserted ahead of each block at the transmitter and removed at the receiver. Transmit functions characteristically include data scrambling, convolutional coding, interleaving, subcarrier modulation mapping, generation of pilot subcarriers, and OFDM modulation using an inverse FFT. Fig. 5 shows an OFDM transmitter algorithm and architecture using FFT-based processing.

Fig 5: OFDM Transmitter Architecture



At the transmitter, the data is coded and interleaved. If there are to be M subcarriers, then baseband processing allows M parallel subcarrier modulation streams to be generated in the frequency domain as complex vectors, each reflecting the amplitude and phase of a subcarrier. Next, an inverse FFT of size  $N \geq M$  converts the complex data from the frequency domain into the time domain effectively modulating the parallel data streams onto M subcarriers. The cyclic prefix is then appended to each symbol prior to digital-to-analog conversion and transmission.

**4.3 OFDM WAVEFORM RECEPTION**

Fig. 6 shows an OFDM receiver algorithm and architecture using FFT-based processing. The FFT-based process is extremely efficient for processing a large number of frequency channels as found in an OFDM waveform. Receive functions typically include automatic gain control (with control interface to the RF section), carrier and clock recovery, OFDM demodulation (using an FFT),

equalization, demapping, deinterleaving, Viterbi decoding, descrambling, and channel assessment and indication to the Medium Access Controller (MAC) layer.

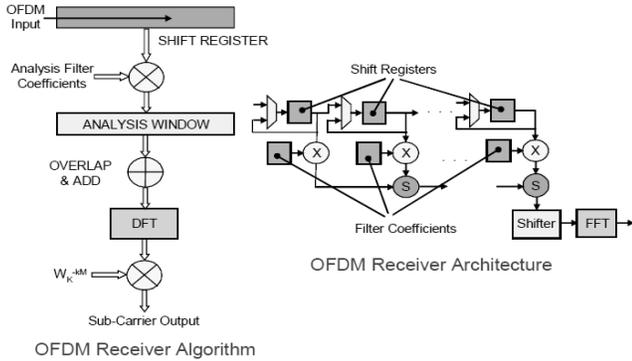


Fig 6: OFDM Receiver Architecture

At the receiver, after down-conversion, analog-to-digital conversion, and removal of the cyclic prefix, then a size N FFT acts as a bank of matched filters to translate the received signal into a parallel stream of  $M \leq N$  complex data representations of the received modulation constellation values for each of the M subcarriers. Equalization for channel distortions, deinterleaving, and decoding results in the receiver's estimate of the transmitted data stream.

5 SIMULATION

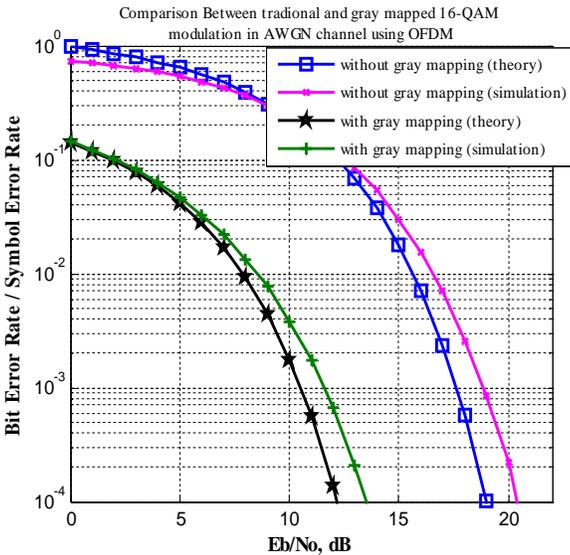


Fig. 7 : SER/BER curve for traditional and gray mapped 16-QAM in AWGN channel

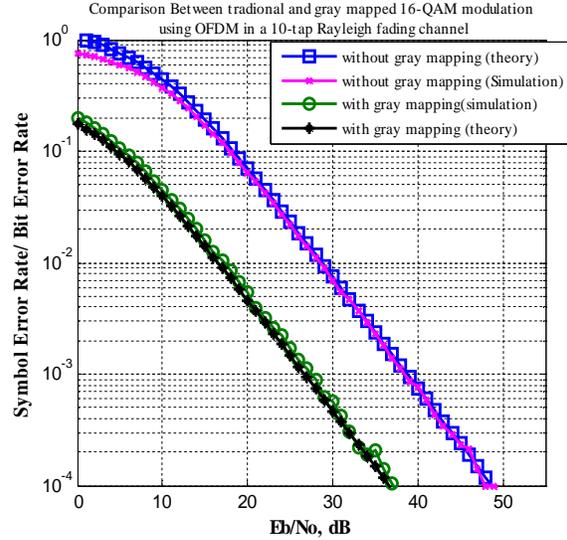


Fig. 8: SER/BER curve for traditional and gray mapped 16-QAM in Rayleigh fading channel

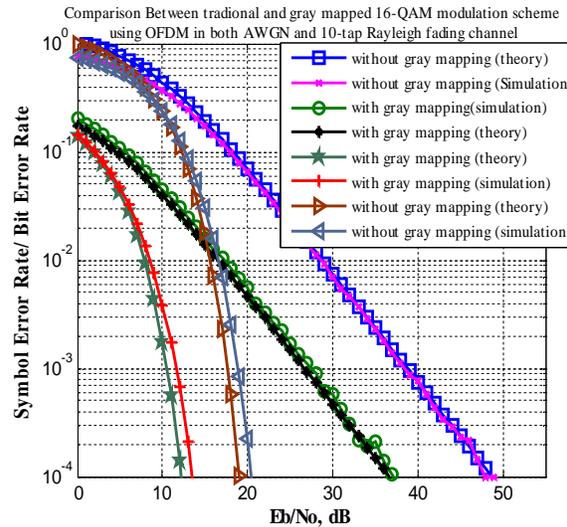


Fig. 9: SER/BER curve for traditional and gray mapped 16-QAM in both AWGN and Rayleigh fading channel

The following figure shows the performance between the traditional and Gray mapped 16-QAM scheme in AWGN channels (Fig. 7). The next figure again presents the performance between the traditional and Gray mapped 16-QAM but this time in a more realistic, Rayleigh fading channel (Fig. 8). Finally we combined both the above mentioned figures together to show an overall comparison (Fig. 9).

6 CONCLUSION

In this paper, we have shown the performance difference between the traditional binary mapped and the gray mapped 16-QAM modulation scheme using OFDM in both AWGN and Rayleigh fading channels. We have found that

the Gray mapped version of 16-QAM has a better SER/BER vs SNR performance, a gain of around 7 dB compared to the traditional one for the AWGN channel and around 10 dB for the Rayleigh Fading channel at  $10^{-4}$  SER/BER.

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