Performance Analysis of Savitzky-Golay Smoothing Filter Using ECG Signal

Md. Abdul Awal, Sheikh Shanawaz Mostafa and Mohiuddin Ahmad

Abstract—Cardiovascular diseases (CVDs) are the most widespread cause of death in many countries all over the world. Electrocardiogram (ECG) is one of the most basic useful, easily available and low cost tools for the early diagnosis and evolution of many cardiac problems. ECG signal can potentially corrupted by various types of noises which lead to incorrect diagnosis. Many types of filters are available for filtering or smoothing the noisy ECG. Some of them are not very much effective and some of them destroy the characteristics of ECG signal during filtering process. Savitzky-Golay (S-G) is one of the filters which can smoothen out the signal without much destroying its original properties. Polynomial degree and frame size are the two parameters of S-G filter and the performance of S-G filter mostly depends on them. The effect of the variation of polynomial degree and frame size are studied in this paper. For denoising ECG signal by using S-G filter PRD and SNR are used as the performance evaluating factor. The experimental results indicate that which type of value of polynomial degree and frame size are better for denoising ECG signal.

Index Terms—CVDs, ECG, noise, S-G filter, polynomial degree, frame size.

1 INTRODUCTION

The heart is one of the major organs of the human body, vital to our survival. It is basically a large pump, whose sole purpose is to maintain blood circulation and keep organs alive. Heart disease has no geographic, gender or socio-economic borders. According to World Health Organization (WHO) 2003 reports CVDs made up 16.7 million, or 29.2% of total global deaths. By 2010, CVD will be the leading cause of death in developing countries. [6]

The autonomous cardiac cell is one of the electrically active cells of human body. Normally they are polarized. These cardiac cells can lose normal negativity by depolarization process. Once the depolarization is complete, the cardiac cells are able to restore their normal polarity by a process called re-polarization. If this electrical activity is recorded in time domain, we get a signal which is called ECG (Fig. 2). Physicians currently use several other diagnostic methods ranging from invasive techniques such as heart catheterization and biochemical markers from blood samples, to non-invasive methods such as computerized tomography (CT) scans and MRI. It is widely accepted that, ECG is one of the reliable and low cost tools for detecting most of the CVDs.

Like other electrical signal, ECG signals are corrupted by various kinds of noise. There are limits to the applicability of the plain FIR average filter for removing noise. In order to achieve a high degree of noiseless signal, its length may be required to be so large that the filter’s pass band becomes smaller than the signal bandwidth. That causes the removal of useful high frequencies from the desired signal. If we consider ECG, \( x(n) = s(n) + v(n) \), where \( s(n) \) is actual ECG signal and \( v(n) \) is the noise. Attempt to smooth out the noise \( v(n) \), the filter begins to smooth out the desired signal \( s(n) \) to an undesirable scale. The S-G FIR smoothing filters, also known as polynomial smoothing, or least-squares smoothing filters are generalizations of the FIR averager filter that can preserve better the high-frequency content of the desired signal.[1]

2 DESCRIPTION OF ECG

Just as the electrical activity of the pacemaker is communicated to the cardiac muscle, echoes of the depolarization and repolarization of the heart are sent through

![Diagram of Electrical activity in heart](image)

The specialized pacemaker cells that start the electrical sequence of depolarization and re-polarization of cardiac tissue is called inherent rhythmicity or automaticity. The electrical signal is generated in the sino-atrial node(SA node)
and spreads to the ventricular muscles via particular conducting pathway: internodal atrial fibers, the atrioventricular node (AV node), the bundle of His, the right and left bundle branch (RBB and LBB), the purkinje fibers then to ventricle (Fig. 1).

By introducing pairs of very sensitive receivers (Electrodes) on other parts of the body, the “echoes” of the heart’s electrical activity can be detected. [2]

This electrical event of the heart is usually on the ECG as a pattern of a baseline (straight line on the ECG, a point of departure of the electrical activity of depolarizations and repolarizations of the cardiac cycles) broken by P wave, QRS Complex and a T wave (Fig. 2). It should be noted that there are some interval (an interval is part of the ECG containing at least one wave and straight line before the QRS complex) and segment (it is the period of time from the end of one wave to the beginning of the next wave. For example, the PR interval includes the P wave and the connection line before the QRS complex) and segment (it is the period of time from the end of one wave to the beginning of the next wave. For example, the PR segment represents the time of AV nodal delay and transmission to the ventricle) between the waves. The generalized properties of ECG are briefly described in table I. [2] [3] [7]

### TABLE I

**DESCRIPTION OF ECG SIGNAL COMPONENTS**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Amp(mV)</th>
<th>Duration</th>
<th>Represents</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.25</td>
<td>0.08</td>
<td>Polarization of Artia</td>
</tr>
<tr>
<td>Q</td>
<td>25% of R</td>
<td></td>
<td>Spetal Depolarization</td>
</tr>
<tr>
<td>R</td>
<td>1.60</td>
<td></td>
<td>Ventricle Depolarization</td>
</tr>
<tr>
<td>P-R Interval</td>
<td>0.12-0.20</td>
<td></td>
<td>Time taken SA node to travel to Ventricle</td>
</tr>
<tr>
<td>QRS Complex</td>
<td>0.09</td>
<td></td>
<td>Ventricle depolarization and Contraction</td>
</tr>
<tr>
<td>T</td>
<td>0.1-0.5</td>
<td>0.16</td>
<td>Beginning of ventricular relaxation</td>
</tr>
<tr>
<td>S-T segment</td>
<td>0.05-0.15</td>
<td></td>
<td>Interval between S and T Wave</td>
</tr>
</tbody>
</table>

### 3 BACKGROUND PROBLEM

Electrocardiographic signals (ECG) may be corrupted by various kinds of noise [4]. Typical examples are:

- Electrical interference from power lines adding 50 or 60 Hz power-line frequency. These include fundamental signals as well as their harmonics with amplitude of up to a maximum of 50% peak-to-peak ECG amplitude.
- Muscle contraction and muscle activity can generate high-frequency electromyography (EMG) noise.
- Motion artifacts such as movement of the electrode over the skin surface.
- Impedance changes at the skin/electrode interface due to temporary loss of contact or loose electrodes.
- Baseline drifts due to respiration.
- Noise introduced due to instrumentation or electronic devices.
- Electrosurgical noise.

Due to this noise, ECG signal is affected extensively such as actual amplitude and duration can changed and lead to erroneous diagnosis. This is the major reason of using filtering process in ECG. So the choice of filter is very essential for removing noise.

### 4 METHODOLOGY

For solving the noise problem of ECG signal S-G filter can be used. It is a simplified method for calculating differentiation and smoothing of data by a least-squares technique. Its computational speed is better than least-squares techniques. One of the major drawbacks: Some of first and last data point cannot smoothen out by the original Savitzky-Golay method. But J. Steinier et. al. and A. Khan improved this method in which this problem is overcome [5].

Assuming that, filter length or frame size (in S-G filter number of data sample read into the state vector at a time) N is odd, N=2M+1 and N ≥ d+1, where d= polynomial order or polynomial degree. [1]

If we have noisy samples $x_n, n = 0, 1, \ldots, L - 1$ and wish to replace them by their smoothed output versions $y_n, n = 0, 1, \ldots, L - 1$ then data vector x have n=L input points and $x=[x_0, x_1, \ldots, x_{L-1}]^{T}$ is replaced by N dimensional one, having M points of each side of $x$

$$x=[x_M, \ldots, x_1, x_0, x_1, \ldots, x_M]^{T}$$ (1)

There are three cases, for calculating the output result. We use case I, Case II, Case III for the simplicity of the calculation.

**Case I**

First M+1 output $y_i$ is calculated by

$$y_i = b^T_{M}, w(M)$$, $i = 0,1 \ldots M$ (2)

Where state vector, w (M) is
\[ w(M) = \begin{bmatrix} x_{N-1} \\ x_{N-2} \\ \vdots \\ x_0 \end{bmatrix} \]  

(3)

Case: II
If \( n \geq L + 1 \) steady output is calculated by setting \( n = L + 1 \).

\[ y_{L-1-M+m} = b^T_m w(L-1-M) \]  

(4)

Where \( m = 0, 1, \ldots, M \) where

\[ w(L-1-M) = \begin{bmatrix} x_{L-1} \\ x_{L-2} \\ \vdots \\ x_{L-N} \end{bmatrix} \]  

(5)

Case: III
Remaning smoothed samples using steady state filter \( b_0 \)

\[ y_n = b_0^T w(n) \]  

(6)

Where

\[ w(n) = \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \vdots \\ x_n \\ x_{n-1} \\ \vdots \\ x_{n-M} \end{bmatrix} \]  

(7)

The coefficient matrix, \( B \) can be determined by the following method.

The S-G filter coefficient \( b_0, b_1, \ldots \) is the element of matrix \( B \),

\[ B = [b_0, b_1, \ldots, b_{N-L}] = GS^T \]  

(8)

Where \( G = S(S^T S)^{-1} \), \( S = [s_0, s_1, \ldots, s_d] \), \( s_0(m) = 1 \), \( s_1(m) = m \), \( s_2(m) = m^2 \), \( \ldots \), \( s_d(m) = m^d \) where \(-d \leq m \leq d\).

For better understanding, if we consider an ECG signal of input data point \( L = 10 \) and quadratic \( (d = 2) \) and frame size \( N = 5 \), then output of S-G smoothing filter can be calculated easily like Fig. 3.

In this example (Fig. 3) frame size \( N = 5 \). From case I, we know that \( M + 1 = 3 \), where \( M = (N-1)/2 \). This three output data point \( y_0, y_1, y_2 \) are calculated by (2).

From case II, last \( M + 1 = 3 \) data point which start from \( (n \geq L - 1 - M) \) th position that means \( y_7, y_8, y_9 \) are calculated by (4) because the they fall in case III. So total output \( y_n \) are calculated by using case I, case II and case III.

Finally, the percentage root mean square difference \((PRD)\) (eqn. 9) and signal to noise ratio \((SNR)\) (eqn.10) are calculated to verify the improvement in the reconstructed signal.

\[ PRD = \sqrt{\frac{\sum_{n=0}^{N} (V(n) - V_R(n))^2}{\sum_{n=0}^{N} V^2(n)}} \times 100\% \]  

(9)

\[ SNR = \log_{10} \frac{\sum_{n=0}^{N} V_R^2(n)}{\sum_{n=0}^{N} S_R^2(n)} \]  

(10)

\( V(n) \): original ECG signal.

\( V_R(n) \): reconstructed ECG signal.

\( S_R(n) \): the deformation in reconstructed ECG signal.
PERFORMANCE ANALYSIS OF SAVITZKY-GOLAY SMOOTHING FILTER USING ECG SIGNAL

5 SIMULATION AND RESULTS

For evaluating the performance of S-G smoothing filter we consider a noisy ECG signal as shown in Fig. 4.

This noisy ECG is the taken as input of S-G filter simulator. We get different results through the implementation of the equation of S-G filter that stated above. There are two parameters, polynomial degree and frame size, as we discussed above. For better understanding the performance of S-G filter, we vary one parameter whereas another is taken as constant. We consider PRD and SNR as a performance evaluating parameter of this filter.

From the simulation (Fig. 5) we can observe that, if the frame size is increased, signal is more smoothen considering the polynomial degree as constant. This phenomenon is seen in the PRD and SNR simulation when frame size is increased (Fig. 6 and Fig. 7).

Because in S-G filter, frame size means number of data points \((2M+1)\) used for smoothen a value. A smoothened value consist of calculating \(M\) data on left and right side. For this reason, increasing frame causes better results.

On the other hand, we also examine that; if polynomial degree of S-G filter is decreased then more noise will be removed (Fig. 8). For the reason that if degree is increased the number of first input-on transient and last input-off transient (Fig. 3) is increased. Because, total input data length is fixed. So number of steady state output that means symmetric filtering is lower. This phenomenon is seen in the PRD and SNR simulation when polynomial degree is decreased (Fig. 9 and Fig. 10).
PERFORMANCE ANALYSIS OF SAVITZKY-GOLAY SMOOTHING FILTER USING ECG SIGNAL

7 CONCLUSION

For detecting CVDs, there is no other techniques is as popular as ECG. As ECG is a bioelectric signal of lower amplitude like other electrical signal it is affected by noise. We tried to reduce this noise problem by using S-G filtering varying polynomial degree and frame size. In this paper we took the PRD and SNR as a performance analyzer factor. It is known to all that, we get better result when PRD is decreased and SNR is increased. From our simulation, better result was found when the polynomial degree was decreased and frame size was increased. If we increase frame size data is more smoothen. Reverse phenomena is true about polynomial degree. In future we will create an adaptive S-G filter which can adaptively select polynomial degree and frame size to tolerate this problem and generate better performance.

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REFERENCES


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